20/2/25

SEM:- IV MJCPHY 06

VECTOR POTENTIAL Unit:-1

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$$\vec{B} = \text{Curl } \vec{A} \qquad \text{--- (1)}$$

Where $\vec{A}$ is a vector function of position, which is the vector potential.

In other words, the vector $\vec{A}$ the ~~curl~~ curl of which is equal to the magnetic induction $\vec{B}$ is known as vector potential

$$B_x\hat{i} + B_y\hat{j} + B_z\hat{k} = \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right)\hat{i} + \left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right)\hat{j} + \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)\hat{k}$$

Now equating the components on both sides of the equation.

$$\left.\begin{aligned} \vec{B}_x &= (\nabla \times \vec{A})_x = \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right) \\ \vec{B}_y &= (\nabla \times \vec{A})_y = \left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right) \\ \vec{B}_z &= (\nabla \times \vec{A})_z = \left(\frac{\partial A_z}{\partial x} - \frac{\partial A_x}{\partial y}\right) \end{aligned}\right\} \text{--- (2)}$$

where A_x, A_y and A_z are the corresponding components of the vector potential A.

Divergence of $\vec{A}$

As div. $\vec{B} = 0$ we have written

$$\vec{B} = \nabla \times \vec{A},$$

i.e. $\nabla \cdot B = \nabla \cdot (\nabla \times \vec{A}) = 0$

The field A is called the vector potential.

In cases where $\nabla \times \vec{B} = 0$, we have

$\vec{B} = -\nabla V_m$, where V_m = scalar potential.

V_m is a multi valued function.

$$V'_m = V_m + c$$

V'_m & V_m represent the same phenomens i.e the same magnetic field, there can be two or more scalar potentials differing by addition of a constant C, since gradient ∇C is zero, $\therefore V'_m = V_m$ represent the same phenomens i.e the same magnetic field.

Similarly

$$\vec{B} = \nabla \times \vec{A} = \nabla \times \vec{A}'$$

$$\therefore \nabla \times \vec{A}' - \nabla \times \vec{A} = \nabla \times (\vec{A}' - \vec{A}) = 0$$

But if the curl of a vector is zero, it must be the gradient of same scalar field, ϕ

So $$A' - A = \nabla \phi$$

$$A' = A + \nabla \phi \qquad (3)$$

will be an equally satisfactory vector potential leading to the same field $\vec{B}$.

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